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QUADRIVIUM IN VARRO'S *DISCIPLINES**

Introduction

Since the time of Archytas, geometry, arithmetic, astronomy, and harmonics had been perceived as kindred μαθήματα (47 B 1 DK).¹ Similar views were shared by Archytas's friend Plato, who considered these sciences a crucial part of the education of the guardians in the ideal state (*Resp.* 521 d – 531 d). We also know of some Pythagoreans and Sophists who taught four μαθήματα, such as Theodorus of Cyrene (*Theaet.* 145 a) and Hippias of Elis (*Prot.* 318 e). Scholars have suggested that the educational curriculum of the *Republic* was not that far from the real one used in the Academy or elsewhere² – overlooking the fact that Plato's model represented an ideal and did not provide an accurate description of any existing curriculum. Plato mostly appreciated μαθήματα as a tool useful for turning guardians' souls to dialectics (*Resp.* 521 c, 532 b–c). His older contemporary Isocrates viewed μαθήματα as a stepping stone on the way to further education, as mathematical subjects were a sort of “gymnastics of the mind” (*Ant.* 261–268). Thus, the quadrivium (both scientific and educational) was born in the fourth century BCE.

Still, the role of μαθήματα in the ancient post-school education³ of the Classical and Hellenistic periods was quite limited, with an emphasis being put instead on rhetoric and philosophy.⁴ Although some teachers of rhetoric and philosophy (namely Isocrates, Aristotle, Xenocrates,⁵ and

* This article was prepared within the framework of Russian Foundation for Basic Research (RFBR) research project No 20-011-00-509.

¹ Huffman 2005, 64; Zhmud 2006, 62–63.

² Cherniss 1945, 66–67; Kühnert 1961, 72–73, 112–117.

³ The post-school mathematical education is understood here as a type of education undertaken after completing the standard school curriculum. It could have been pursued for its own sake or, more commonly, as προπαιδευμα to other studies, such as law, rhetoric, philosophy, etc.

⁴ Marrou 1964; Clarke 1971; Bonner 1977; Barrow 2015, 286–289.

⁵ DL 4. 10: “To someone who had never learnt music, geometry, or astronomy, but nevertheless wished to attend his lectures, Xenocrates said: ‘Go your ways, for you offer philosophy nothing to lay hold of’” (tr. Hicks 1925, 385).

Arcesilaus⁶) considered a certain level of knowledge in mathematical subjects useful, there is still no positive evidence to suggest that during the Hellenistic period the mathematical sciences were taught on a regular basis not only to specialists, but also to a wider audience. However, H.-I. Marrou has supported the view that the circle of the seven *artes liberales* had already been formed in the Hellenistic period.⁷ At some point during the Hellenistic period, there must have appeared a mathematical education directed not only at professionals, as suggested by the existence of popular introductions into mathematical disciplines, e.g. Geminus' *Introduction to the Phenomena* (first century BCE). The first Roman author to document this change was the encyclopedic writer Varro (first century BCE), in whose writings traces of the Greek tradition can be found.

Varro was most probably the first to have unified what later became known as the trivium (grammar, dialectic, rhetoric⁸) and the quadrivium (geometry, arithmetic, astronomy and harmonics) in one now lost encyclopedic work. Varro's *Disciplines* has been regarded as a link between Greek and Roman educational practices. Hellfried Dahlmann has described it as the "vielleicht einflußreichste Werk Varros", arguing that the Greek *artes liberales* did not have a constant number and order, and that Varro's major achievement was to define them.⁹ Meanwhile, according to Friedmar Kühnert, all later encyclopedias depended on Varro, whether directly or indirectly.¹⁰ Since very little is known of the content of Varro's *Disciplines*, it is not clear whether the work was intended to be a coursebook for use in a classroom, or rather a popular book for self-educational purposes. The word *disciplina* itself denotes instruction and teaching in the widest sense of the word; metonymically it can also mean all that is taught in the way of instruction.¹¹

In late Antiquity, under the influence of Varro's *Disciplines*, the quadrivium was incorporated into the works of Augustine,¹² Cassiodorus,¹³

⁶ Arcesilaus himself was a pupil of the mathematician Autolycus (DL 4. 29), the musician Xanthus, and the geometer Hipponicus (*ib.* 32). Moreover, he was annoyed with any who took up their studies too late (*ib.* 36).

⁷ Marrou 1969, 12.

⁸ Grammar, rhetoric, and dialectic as an educational unity first appear among the Stoics, who considered them components of logic.

⁹ Dahlmann 1935, 1255; 1257.

¹⁰ Kühnert 1961, 67.

¹¹ See Lewis–Short 1879, 587 s.v. *disciplina*.

¹² D'Alessandro 1997, 357–370; Shanzer 2005, 69–95; Gasti 2017, 303–318.

¹³ D'Alessandro 1997; Schindel 2006, 99–108.

Martianus Capella,¹⁴ and Isidore of Seville,¹⁵ and hence made its way into medieval pedagogy. Reconstructing the work's content and book order was attempted, among others, by Friedrich Wilhelm Ritschl,¹⁶ whose influence on later traditions is hard to overestimate.¹⁷ However, Ritschl's reconstruction, together with Marrou's thesis that the circle of seven liberal arts was already extant in the Hellenistic period, have been severely criticized by Ilsetraut Hadot, who has argued that the circle of liberal arts came into being in Neoplatonic circles and was then transmitted to the Medieval West through Augustine's works. To prove this, Hadot greatly underplayed the role of Varro's *Disciplines*, claiming that "we know almost nothing about its content and order" and that "we do not know whether some of the books were devoted to mathematical sciences – if at all".¹⁸ In a more recent article, Hadot claims: "Les sources littéraires et épigraphiques relatives à l'enseignement habituel ou ordinaire dans les gymnases hellénistiques et les écoles privées ne parlent jamais d'un enseignement régulier en sciences mathématiques".¹⁹ She holds the same view on mathematical education in the Imperial period. Within this dispute, I would like to take the middle ground. While the scarcity of Greek evidence does not allow for far-reaching conclusions, we are already in Varro – who was undoubtedly well acquainted with and made extensive use of Greek sources – faced with clear and convincing evidence contradicting Hadot's conclusions.

The goals of this paper are therefore (1) to defend the tradition positing that Varro's *Disciplines* did include four books devoted to the subjects of quadrivium; (2) to provide a description of its content, composition, and sources; and (3) to attempt an account of mathematical education in Varro's time.

Literary evidence suggests that in some circles of Roman society, there was a certain, though modest, demand for mathematical education. For those in the higher circles of society, there was a constant pressure to be (or seem) sufficiently educated, and μαθήματα were among the subjects one needed to be familiar with. Varro's *Disciplines* was probably an entry-level book that provided the reader with useful and concise information about all the subjects considered necessary for a noble and well-educated person.

¹⁴ Stahl 1971.

¹⁵ Barney et al. 2006.

¹⁶ Ritschl 1877, 352–402.

¹⁷ Dahlmann 1935, 1255; Fuchs 1962, 387; Kühnert 1961, 58 ff.; Simon 1966, 94.

¹⁸ Hadot 1984, 156; 168.

¹⁹ Hadot 1998, 233–250.

Most of our evidence comes from Cicero, according to whom some people preferred to spend their free time indulging in geometry (*De orat.* 3. 58). These include a Roman general, Sextus Pompeius, who was quite successful in his studies of geometry (*Brutus* 175 and *De off.* 1. 19), and the poet and politician Cornelius Gallus, who devoted himself *studio dimetiendi paene caeli atque terrae* (*De sen.* 49). A Stoic philosopher, Diodotus, spent many years living under Cicero's roof. Despite being blind, Diodotus continued teaching geometry: *quod sine oculis fieri posse vix videtur, geometriae munus tuebatur verbis praecipiens discipulis, unde quo quamque lineam scriberent* (*Tusc.* 5. 113). Cicero himself was his student in dialectics but also in "many other things" (*Brutus* 309). Cicero's interest in astronomy is well-known; he even translated Aratus's poem *Phaenomena*. Furthermore, Suetonius writes that Vergil *maxime mathematicae operam dedit* (*Vit. Verg.* 15), while Caesar gave Roman citizenship to *liberalium atrium doctores, quo libentius et ipsi urbem incolerent et ceteri adpeterent* (*Caes.* 1. 42. 1). Plutarch mentions that Pompeius's wife was well-versed in geometry (*Pomp.* 55). In the first century AD, Columella, lamenting the terrible state of agricultural education, mentions that people are extremely careful in choosing their teachers in a number of different disciplines – including geometry – and that there even exist *scholae geometrarum* (1. *Praef.* 5). In fact, schools owned by private teachers must have existed even earlier: the aforementioned Diodotus had what was essentially a *schola*.²⁰ Quintilian insisted that *nullo modo sine geometria esse possit orator* (*Inst.* 1. 49).

The evidence for studying geometry and other mathematical sciences as a part of higher education in Varro's time is far from abundant, but we still can conclude that some people who did not in the slightest aspire to become professional mathematicians, architects, and so on, did indeed have some experience in studying the μαθήματα.

Though authors mentioning Varro in relation to the μαθήματα are not numerous, the evidence certainly appears direct and clear in support of Varro's involvement in the μαθήματα:

1) Pliny draws a lot of his geographical evidence (especially, measurements) from Varro (*NH* 3. 45, 95, 109; 4. 77–78, 115–116).

2) From Aulus Gellius we know that Varro made some observations about a certain *ratio geometrica* in one of the books of the *Disciplines* (*NA* 18. 15), wrote about different parts of geometry (16. 18), and provided geometrical definitions (1. 20).

²⁰ On Roman *scholae*, see Bonner 1977.

3) Augustine's friend Licentius of Tagaste in his *Carmen ad Augustinum* portrays himself toiling over Varro's books – *perplexa viri compendia tanti* (5). In the following lines, he makes references to the studies of music (7–8), geometry (11–12), and astronomy (13–14).²¹

4) Cassiodorus attests to the existence of both *volumen geometriae* (*Inst.* 2. 7. 4) and *liber de astrologia* (*ib.* 2). Moreover, Varro is one of the sources for Cassiodorus's book on music (*ib.* 5. 8). Varro's views on the origin of geometry are also reported by Cassiodorus (*ib.* 6. 1).

5) Martianus Capella refers to Varro several times: in his geometry book in *De nuptiis*, he says that geometry crossed the thresholds of very few Romans, among them Varro (6. 578); Capella then mentions the latter in two geographical measurements (6. 639, 662), in his books on astronomy (8. 817) and music (9. 928). Even more numerous are his allusions to Varro.²²

6) Claudianus Mamertus is familiar with Varro's books on music and geometry (*De statu animae* 2. 8).

Thus, our evidence comes from a number of authors and explicitly indicates not only the mere existence of books on different mathematical subjects, but in the case of geometry – where the evidence is most ample – it also outlines the book's content, as we will see later.

Geometry: its origin and connection to astronomy

Let us start with Varro's views on the origin of geometry and how they relate to the ancient tradition. To do so, we are going to examine a text that exists in four versions. The earliest surviving version is found in *Institutiones* by Cassiodorus (approximately mid-530s or later),²³ while the other three are Isidorus's *Etymologiae*, written between 615 and 632, Pseudo-Boethius's *Demonstratio artis geometriae* (not later than the eighth century),²⁴ and an anonymous treatise titled *De septem artibus liberalibus* from the eighth century.²⁵ Of these, Cassiodorus's provides the fullest account, and after Lachmann's edition of Pseudo-Boethius, it was long believed that the latter's text depended directly on Cassiodorus. Schindel, however, demonstrated that all texts including the last treatise, which he

²¹ For a critical text accompanied by an insightful commentary, see Shanzer 1991, 110–143.

²² See index to Stahl 1971, s.v. *Varro*.

²³ Halporn–Vessey 2004, 23–24.

²⁴ Lachmann 1848, 393–406.

²⁵ Schindel 2004, 132–144.

was the first to publish, go back to a common source which Schindel dates ca. 500 CE.²⁶ Quite tellingly, all of our sources are encyclopedic and were written with educational purposes in mind. Cass. *Inst.* 2. 6. 1:

Geometria latine dicitur terrae dimensio, quoniam per diversas formas ipsius disciplinae, ut nonnulli dicunt, primum Aegyptus dominis propriis fertur esse partitus; cuius disciplinae magistri mensores ante dicebantur. sed Varro, peritissimus Latinorum, huius nominis causam sic exitissime commemorat, dicens prius quidem dimensiones terrarum terminis positus vagantibus ac discordantibus populis pacis utilia praestitisse; deinde totius anni circulum menstruali numero fuisse partitum, unde et ipsi menses, quod annum metiantur, edicti sunt. verum postquam ista reperta sunt, provocati studiosi ad illa invisibilia cognoscenda coeperunt quaerere quanto spatio a terra luna, a luna sol ipse distaret, et usque ad verticem caeli quanta se mensura distenderet; quod peritissimos geometras assecutos esse commemorat. tunc et dimensionem universae terrae probabiliter refert ratione collectam; ideoque factum est ut disciplina ipsa Geometria nomen acciperet, quod per saecula longa custodit.

Geometry in Latin means the measurement of the earth; some say it is so named because Egypt was first divided among its own lords by various forms of this discipline. In earlier times the teachers of this discipline were called measurers. But Varro, the most learned of the Latin writers, offers the following reason for the name. First the measurement of the earth gave useful peace to wandering peoples [who disagreed] by setting down boundary stones. Then the circle of the whole year was apportioned out by the measurement of the months. As a result, the months themselves were so named because they measure the years. But after these things were discovered, scholars were moved to study intangible phenomena, and began to ask how far the moon was from the earth and the sun from the moon and how far it was to the top of the heavens. He reports that the most learned geometricians arrived at the measurements of these distances. Then he also relates that the measurement of the whole earth was arrived at by a praiseworthy reasoning; thus it came about that the discipline received the name geometry [of geometry] that it bears over the course of the ages.²⁷

Cassiodorus starts his account with a reference to a well-known Greek tradition tracing the invention of geometry to Egypt. According to this tradition, which makes its first appearance in Herodotus, the need

²⁶ Schindel 2006, 99–108.

²⁷ Tr. W. Halporn (Halporn–Vessey 2004, 223).

for the first land surveyors was born due to the annual inundation of the Nile, by which the land was diminished, so the amount of taxes had to be adjusted according to the new land size (2. 109). Aristotle also believes that the exact sciences are of Egyptian origin, but initially related neither to pleasure nor to utility as in Democritus (*Met.* 981 b):²⁸

When all discoveries of this kind (i. e. aimed at utility or pleasure) were fully developed, the sciences which relate neither to pleasure nor yet to the necessities of life were invented, and first in those places where men had leisure. Thus, the mathematical sciences originated in the neighborhood of Egypt, because there the priestly class was allowed leisure.

His student Eudemus of Rhodes considered geometry an Egyptian invention that appeared for the practical purposes of land surveying (Procl. *In Eucl.* 64. 16 = fr. 133 W.).

Varro's view on the matter is quite different. Egypt is not even mentioned in his account; instead, he highlights the role that land-measuring played in the making of human civilization: at first there were some wandering and quarreling tribes (*populi vagantes ac discordantes*) who made peace due to land surveying. It follows that the same invention put an end to their nomadic lifestyle as well and hence *populi vagantes* became settled, although this is not mentioned in the fragment. Before Varro, a similar pacifying role was ascribed by Archytas to arithmetic (47 B 3 DK): the invention of counting put an end to discord (στάσις) and increased concord (ὁμόνοια). Both Varro and Archytas see the inventions' utility (*utilitas*, χρήσιμον) in these social consequences. Moreover, according to Varro, the beginnings of all arts first appeared because of some utility (just as in Democritus): *Scire autem debemus, sicut Varro dicit, utilitatis alicuius causa omnium artium exitisse principia* (Cass. *Inst.* 528).

Having established the origin of geometry, Varro proceeds to how it was used afterwards. According to him, its next contribution was to the calendar, when the year was divided into months: *deinde totius anni circulum menstruali numero fuisse partitum, unde et ipsi menses, quod annum metiantur, edicti sunt*. The fact that Varro assigns geometry responsibility for calendars is quite unusual. Normally, calendars were perceived as a part of astronomy, as they result from the observation

²⁸ Tr. Tredennick 1933, 9. For Democritus and his influence on later tradition v. Cole 1967.

of celestial bodies and their movements. This will not be the last time Varro's geometry appears instead of other sciences.

It is known that Varro himself was interested in calendars: in his *Res rusticae*, he describes the beginning and the duration of each season and gives instructions as to when certain agricultural works have to be executed (1. 28–36). Traditionally, the year's division into months has also been ascribed to Egyptians: Herodotus says that “the Egyptians were the first men who reckoned by years and made the year consist of twelve divisions of the seasons. <...> the Egyptians, reckoning thirty days to each of the twelve months, add five days in every year over and above the total” (2. 4. 1).²⁹ Herodotus's belief in the Egyptian provenance of geometry and year division became a recurring *topos* in ancient thought.³⁰

Interestingly, Varro ascribes these two incredibly important inventions to neither a nation (e.g. Egyptians) nor an individual ingenious inventor (the so-called *πρῶτος εὐρητής*) – nor to philosophy, as his older contemporary Posidonius did (*Sen. Ep.* XC = F 284 E–K). Instead, Varro's views seem to represent an evolution of the tradition that the origin of arts and sciences lies in necessity. Contrary to an almost unanimous opinion that geometry originated in Egypt, Varro traces its invention back to the dawn of human civilization.

In the next sentences, Varro refers to some famous astronomical discoveries that were made through the application of geometry (*provocati studiosi ad illa invisibilia* etc.). Thus, he calls people who made them ‘geometers’. Several Hellenistic scientists are known to have dealt with measurements of distances between the Earth and other celestial bodies. The first scientific attempt was made by Aristarchus of Samos, who claimed that the distance between the Earth and the Sun was about 18 to 20 times bigger than the distance between the Earth and the Moon (*De magn.*). Archimedes and Hipparchus were also interested in the same question. The Earth's circumference (*dimensio universae terrae*) was first calculated by Eratosthenes in his book *Περὶ ἀναμετρήσεως τῆς γῆς*, while Posidonius studied both matters too (Cleomedes 1. 10). Even though Cassiodorus does not give any names, there is still a reason to suggest that Varro did: Cassiodorus's remark “*quod peritissimos geometras assecutos esse commemorat (sc. Varro)*” might mean just that. Hence,

²⁹ Tr. Godley 1920, 279.

³⁰ On the Egyptian provenance of geometry cf. Diod. 1. 69. 5; 81. 3; 94. 3; Strabo 17. 1. 3 (C 788); on Egyptians discovering the year cf. Diod. 1. 50. 1–2; Strabo 17. 1. 29 (C 806); 1. 46 (C 816).

the names must have been left out by an intermediary source. As to why Varro does not draw a distinction between geometers and astronomers, the answer might be that he underlines the primacy of geometry in regard to astronomy, as the latter heavily relies on geometrical principles. Varro might have used geometry as an umbrella term for mathematics.³¹

It appears, then, that we deal with an extremely curtailed version of what used to be a historical introduction to the book of geometry. Is it possible to amplify evidence provided by Cassiodorus, using later sources? Isidorus's summary (*Etym.* 3. 10) is quite brief, but still yields some additional information (in bold):

Geometriae disciplina primum ab Aegyptiis reperta dicitur, quod, inundante Nilo et omnium possessionibus limo obductis, initium terrae dividendae per lineas et mensuras nomen arti dedit. **Quae deinde longius acumine sapientium profecta et maris et caeli et aeris spatia metuntur.** Nam provocati studio sic coeperunt **post terrae dimensionem** et caeli spatia quaerere: quanto intervallo luna a terris, a luna sol ipse distaret, et usque ad verticem caeli quanta se mensura distenderet, sicque **intervalla ipsa caeli orbisque ambitum** per numerum stadiorum ratione probabili distinxerunt. Sed quia ex terrae dimensione haec disciplina coepit, ex initio sui et nomen servavit. Nam geometria de terra et de mensura nuncupata est. Terra enim Graece GE vocatur, METRA mensura. Huius disciplinae ars continet in se lineamenta, intervalla, magnitudines et figuras, et in figuris dimensiones et numeros.

It is said that the discipline of geometry was first discovered by the Egyptians, because, when the Nile River flooded and everyone's possessions were covered with mud, the onset of dividing the earth by means of lines and measures gave a name to the skill. **And thereupon, when it was greatly perfected by the acumen of wise men, the expanses of the sea, sky, and air were measured.** Stimulated by their zeal, these sages began, **after they had measured the land**, to inquire about the region of the sky, as to how far the moon is from the earth, and even the sun from the moon; and how great a distance there is to the pinnacle of the heavens. And so, using reasoning capable of being tested and proved, they determined **the distances of the vault of heaven and**

³¹ Quintilian, in the first book of *Institutio oratoria*, points out that geometry might be useful for an orator in settling land disputes, but also in understanding certain astronomical matters. According to Quintilian, "*quid quod se eadem geometria tollit ad rationem usque mundi? in qua, cum siderum certos constitutosque cursus numeris docet, discimus nihil esse inordinatum atque fortuitum; quod ipsum nonnunquam pertinere ad oratorem potest?*" (1. 10. 46). Another example of a similar approach is found in Cass. *Variae*, 3. 52: (*Geometria*), *quae tantum de caelestibus disputat.*

the perimeter of the earth in terms of the number of stadia. But because the discipline began with measuring the earth, it retained its name from its origin, for geometry (*geometria*) takes its name from “earth” and “measure.” In Greek, “earth” is called GE and “measure” is METRA. The art of this discipline is concerned with lines, distances, sizes and shapes, and the dimensions and numbers found in shapes.³²

As we see, Varro's rather untraditional account of the origin of geometry is completely left out here, which probably explains why a reference to him is missing. Still, the new sentence (*quae deinde longius* etc.) helps us to connect the dots from such simple things as land measurements and the division of the year into twelve months on the one hand to quite advanced astronomical measurements on the other. Isidorus makes it clear that at first the Greeks succeeded in the realm of visible objects by measuring the *maris et caeli et aëris spatia*, which inspired them to turn their attention to the *invisibilia*. Thus, the questions that we have to ask ourselves are the following: what are the distances that are being referred to, and is there any evidence related to them that we can trace back to Varro? When it comes to *spatia maris*, in Martianus Capella's book on geometry there is indeed some evidence that connects Varro with sea measurements (*De nuptiis* 6. 662):

circuitus vero totius Ponti vicies semel quinquaginta milibus, ut Varro quoque non reticet, qui adicit Europae totius longitudinem habere sexagies ter triginta septem milia passusque quingentos.

Moreover, geography seems to have occupied quite a significant place in Varro's book.³³ Both Pliny (3. 45, 95, 109; 4. 77–78, 115–116) and Martianus Capella (6. 662, 639) mention Varro as their source for various geographical measurements.

³² Tr. Barney et al. 2006, 93.

³³ Cf. Strabo (based probably on Posidonius): “Those who write on the science of Geography should trust entirely for the arrangement of the subject they are engaged on to the geometers, who have measured the whole earth; they in their turn to astronomers; and these again to natural philosophers. Now natural philosophy is one of the perfect sciences. <...> Admitting these points in whole or in part, astronomers proceed to treat of other matters, [such as] the motions [of the stars], their revolutions, eclipses, size, relative distance, and a thousand similar particulars. On their side, geometers, when measuring the size of the entire earth, avail themselves of the data furnished by the natural philosopher and astronomer; and the geographer on his part makes use of those of the geometer” (Geogr. 2. 5. 2; tr. Hamilton–Falconer 1853, 166–167). In Varro's hierarchy, as opposed to this one, natural philosophy does not seem to claim primacy over sciences.

Spatia caeli et aëris, however, present some problems: first of all, at first glance, *spatia caeli* seem to be out of place here, as in the next sentence we have *coeperunt post terrae dimensionem et caeli spatia quaerere: quanto intervallo luna a terris* etc.: so, *spatia caeli* are astronomical measurements that were carried out by scientists after they measured *terrae dimensionem*. Secondly, what are *spatia aëris*?

I suggest considering an example of what could have been described as *spatia aëris*. Varro's older contemporary Posidonius "supposes that there is a space of not less than 40 stadia around the earth, whence mists, winds, and clouds (*nubila ac venti nubesque*) proceed".³⁴ So, there might be no need to get rid of the first *spatia caeli* in *spatia caeli et aëris*: one could understand them together as the distances between the earth, clouds, and air that were measured before scientists ventured even further to measure celestial distances.

The ascent from the visible to the invisible might have been a part of the same tradition that Varro used, which described the emergence of sciences due to necessity and their gradual evolution from the practical to the more and more abstract.³⁵ This ascent has been interpreted by some³⁶ as purely philosophical (leading some scholars even further to conclude that there was a book on philosophy in the *Disciplinarum libri* instead of one on astronomy).³⁷ That is simply not the case: the existence of his astronomy book is attested to by different sources³⁸ (while that of a book on philosophy is not) and both the *visibilia* and *invisibilia* in his account are strictly scientific. *Spatio maris* and *dimensio terrae* are named among the achievements of geometers in the realm of *visibilia* – while such unmistakably astronomical questions as *quanto intervallo luna a terris, a luna sol ipse distaret, et usque ad verticem caeli quanta*

³⁴ Plin. *NH* 2. 21. Further he writes: "Beyond this, he supposes that the air (*aëra*) is pure and liquid, consisting of uninterrupted light; from the clouded region to the moon there is a space of 2 000 000 of stadia, and thence to the sun of 500 000 000. It is in consequence of this space that the sun, notwithstanding his immense magnitude, does not burn the earth. Many persons have imagined that the clouds rise to the height of 900 stadia" (tr. Bostock–Riley 1855, 53–54). There had been a vivid discussion of the cloud belt since Aristotle (*Meteor.* 340 a–b). See F 120 E–K.

³⁵ Cf. Pizzani 1976, 460.

³⁶ Fuchs 1926, 158 n. 1, followed by Dahlmann 1935, 1258, suggested that in the distribution of material, Varro followed a spiritual itinerary from corporeal to incorporeal similar to the one in Augustine.

³⁷ Della Corte 1954, 239, 247–253; Salignac 1958, 122–123; Pizzani 1974, 672–675.

³⁸ V. the introduction to this article.

se mensura distenderet, were, according to Varro, also answered by *geometrae*. In fact, the 'visible to invisible' ascent does not have to always be Neoplatonic in origin. Here, it might be a part of a scientific method used in exact sciences, where the visible serves as a starting point for the cognition of the invisible.³⁹

Thus, the Geometry book of Varro's *Disciplines* must have started with a historical account of the birth of geometry as the art of land-surveying, in which Varro (or his source) offers quite a unique perspective on the origin of geometry and the role it played in the history of humankind. It would have been followed by a list of scientists and their geographical and astronomical discoveries. It is reasonable to suppose that introductions to other books of *Disciplinae* followed the same structure: the invention of a discipline due to a certain utility and a list of scientists and their discoveries. Thus, arithmetic might have been invented because it is useful in trade (as stated in Eudemos), astronomy because it made navigation easier, and music because it – since the time of Pythagoras – was thought to have a calming effect on people and animals (Cass. *Inst.* 2. 5. 8):

Unde claret quoniam hyperlydius tonus omnium acutissimus septem tonis praecedit hypodorium omnium gravissimum. In quibus, ut Varro meminit, tantae utilitatis virtus ostensa est ut excitatos animos sedarent, ipsas quoque bestias, necnon et serpentes, volucres atque delfinas ad auditum suae modulationis attraherent.

Augustine's *De ordine* (2. 12. 35) seems to contain traces of a similarly structured introduction to Varro's book of grammar from the *Disciplines*: *quibus duobus repertis* (sc. *litteris et numeratione*) *nata est illa librariorum et calculonum professio velut quaedam grammaticae infantia, quam Varro litterationem vocat*. Here, a discipline again comes into being because of some utility and quickly becomes a specialized field of knowledge with people choosing it as their profession; then, it becomes more and more abstract and detached from day-to-day issues.

³⁹ On this method, that O. Regenbogen called "eine Methode naturwissenschaftlicher Hypothesenbildung durch Analogieschlüsse", see Regenbogen 1930, 131 ff. and Diller 1932, 14–42.

Varro's definitions and Euclid (Gell. *NA* 1. 20)

Aulus Gellius turns to geometry thrice: in the first book of *Noctes Atticae* (1. 20), he offers some geometrical definitions (on which we are going to focus in this section); in the following books he discusses various parts of geometry (16. 18), and notes the relation between a certain poetic meter and a geometric ratio (18. 15). Given that in all three places Gellius mentions Varro as his source, it is safe to say that Varro was his only source on geometry.

Now, to how Aulus Gellius himself viewed the audience he was writing for and what sources he used for his writing, as these directly affect how sophisticated the book's content will prove: in the *Prooemium* to his *Noctes Atticae*, Gellius compares himself to the authors of the same genre, finding that many authors, especially of Greek origin, "swept together whatever they had found, aiming at mere quantity. The perusal of such collections will exhaust the mind through weariness or disgust".⁴⁰ Gellius, on the contrary,

took few items from them, confining myself to those which, by furnishing a quick and easy short-cut, might lead active and alert minds to a desire for independent learning and to the study of the useful arts, or would save those who are already fully occupied with the other duties of life from an ignorance of words and things which is assuredly shameful and boorish (*Pr.* 12).

He makes a special reference to some "obscure" subjects, geometry included:

Now just because there will be found in these notes some few topics that are knotty and troublesome, either from Grammar or Dialectics or even from Geometry, and because there will also be some little material of a somewhat recondite character about augural or pontifical law, one ought not therefore to avoid such topics as useless to know or difficult to comprehend. For I have not made an excessively deep and obscure investigation of the intricacies of these questions, but I have presented the first fruits, so to say, and a kind of foretaste of the liberal arts; and never to have heard of these, or come in contact with them, is at least unbecoming, if not positively harmful, for a man with even an ordinary education (*Pr.* 13).

⁴⁰ Tr. Rolfe 1927 here and for all later quotations from *Noctes Atticae*.

Now we know what to expect from Gellius's notes on geometry: they are supposed to be entry-level, i.e. not difficult to comprehend, but at the same time useful, as not knowing such things is shameful for an educated person. The fact that Gellius had to warn his readers about some "knotty and troublesome topics" pertaining to the fields of grammar, dialectics, and geometry shows exactly the kind of reputation these subjects had: knotty, troublesome, and obscure. Gellius did not go as far as explaining all the intricacies of the aforementioned disciplines, but instead saw himself "pointing out of the path [that] they [i.e. the readers] may afterwards follow up [on] those subjects, if they so desire, with the aid either of books or of teachers" (*Pr.* 17).

Now, let us take a closer look at the geometrical definitions from the first book of *Noctes Atticae*:

On what the geometers call ἐπίπεδος, στερεός, κύβος and γραμμή, with the Latin equivalents for all these terms of the figures which the geometers call σχήματα there are two kinds, 'plane' and 'solid'. These the Greeks themselves call respectively ἐπίπεδος and στερεός. A 'plane' figure is one that has all its lines in two dimensions only, breadth and length; for example, triangles and squares, which are drawn on a flat surface without height. We have a 'solid' figure, when its several lines do not produce merely length and breadth in a plane, but are raised so as to produce height also; such are in general the triangular columns which they call 'pyramids', or those which are bounded on all sides by squares, such as the Greeks call κύβοι, and we *quadrantalia*. For the κύβος is a figure which is square on all its sides, "like the dice", says Marcus Varro, "with which we play on a gaming-board, for which reason the dice themselves are called κύβοι". Similarly, in numbers too the term κύβος is used, when every factor consisting of the same number is equally resolved into the cube number itself, as is the case when three is taken three times and the resulting number itself is then trebled. Pythagoras declared that the cube of the number three controls the course of the moon, since the moon passes through its orbit in twenty-seven days, and the *ternio*, or 'triad', which the Greeks call τριάς, when cubed makes twenty-seven. Furthermore, our geometers apply the term *linea*, or 'line', to what the Greeks call γραμμή. This is defined by Marcus Varro as follows: "A line", says he, "is length without breadth or height". But Euclid says more tersely, omitting "height": "A line is μήκος ἀπλατές, or 'breadthless length'". ἀπλατές cannot be expressed in Latin by a single word, unless you should venture to coin the term *inlatabile*.

Ritschl suggested that Gellius took from Varro not only the two definitions in which Varro is explicitly mentioned (i.e. cube and line), but

the whole paragraph.⁴¹ This is highly probable, given that Varro seems to have been his only source on geometry.

The first thing that immediately catches the eye is the structure of this fragment and its logic (or sometimes lack thereof): the first two definitions (of plane and solid figures) already make use of a term ‘line’, which should have been explained earlier, but instead makes its appearance only at the end of the passage. Still, we are going to look into these definitions in the order in which Aulus Gellius placed them.

a) Plane and Solid Figures

Figurarum, quae σχήματα geometrae appellant, genera sunt duo, ‘planum’ et ‘solidum’. Haec ipsi vocant ἐπίπεδον καὶ στερεόν. ‘Planum’ est, quod in duas partis solum lineas habet, qua latum est et qua longum: qualia sunt triquetra et quadrata, quae in area fiunt, sine altitudine. ‘Solidum’ est, quando non longitudines modo et latitudines planas numeri linearum efficiunt, sed etiam extollunt altitudines quales sunt ferme metae triangulae, quas ‘pyramidas’ appellant, vel qualia sunt quadrata undique, quae κύβους illi, nos *quadrantalia* dicimus.

The definition of a plane figure conveys the meaning of Euclid’s definition,⁴² but does so in a less abstract way: the definition creates visual images of drawn geometrical figures (*in area fiunt*), while the examples of triangles and squares should facilitate understandings of the term. The definition of a solid figure follows the same pattern as that of a plane figure, including the use of examples (cf. Euclid: στερεόν ἐστὶ τὸ μήκος καὶ πλάτος καὶ βάθος ἔχον; *Def.* 11. 1).

b) Cube

<...> vel qualia sunt quadrata undique, quae κύβους illi, nos *quadrantalia* dicimus. Κύβος enim est figura ex omni latere quadrata, “quales sunt”, inquit M. Varro, “tesseræ, quibus in alveolo luditur, ex quo ipsae quoque appellatae κύβοι”. In numeris etiam similiter κύβος dicitur, cum omne latus eiusdem numeri aequabiliter in sese solvitur, sicuti fit, cum ter terna ducuntur atque ipse numerus triplicatur. Huius numeri cubum Pythagoras vim habere lunaris circuli dixit, quod et luna orbem suum lustret septem et viginti diebus et numerus ternio, qui τριάς Graece dicitur, tantundem efficiat in cubo.

⁴¹ Ritschl 1877, 386.

⁴² ἐπίπεδος ἐπιφανεία ἐστὶν ἢ τις ἐξ ἴσου ταῖς ἐφ’ ἑαυτῆς εὐθείαις κείται (*Def.* 1. 7).

The term *quadrantal* is quite interesting. Gellius says that what Greeks call κύβος, Romans call *quadrantal*, but in fact the Latin term in its geometrical meaning occurs only here. Usually, this term is used for volume units.⁴³ In a couple of later authors, this word is also documented in its arithmetical meaning, i.e. arithmetical cube.⁴⁴ What are we to make of this? Seeing that Gellius's only geometrical source was Varro, and Varro himself was one of the first Romans to write on the subject of geometry in Latin, it might well be that it was Varro who "reinvented" the term *quadrantal* that was previously applied only to volume units. Still, the term did not quite catch on, and the Greek *cybus/cubus* became the standard terms.

According to Varro, "the κύβος is a figure which is square on all its sides". A quick comparison to Euclid's definition (κύβος ἐστὶ σχῆμα στερεὸν ὑπὸ ἑξ τετραγώνων ἴσων περιεχόμενον: *Def.* 11. 25) shows that the Varro's one is quite imprecise: it fails to mention the number of squares that are needed to make a cube, as well as the fact that all the squares have to be equal.

The geometrical definition of a cube is followed by an arithmetical one, i.e. "in numbers too the term κύβος is used, when every factor consisting of the same number is equally resolved into the cube number itself". The definition is accompanied by an example of three cubed (cf. *Euc. Def.* 7. 20: Κύβος δὲ ὁ ἰσάκις ἴσος ἰσάκις ἢ [ὁ] ὑπὸ τριῶν ἴσων ἀριθμῶν περιεχόμενος).

The following sentence turns to arithmology, invoking the authority of Pythagoras: "Pythagoras declared that the cube of the number three controls the course of the moon, since the moon passes through its orbit in twenty-seven days, and the ternio, or 'triad', which the Greeks call τριάς, when cubed makes twenty-seven". Considering Varro's interest in arithmology, this passage is hardly surprising. Varro took a great interest in Neopythagoreanism and authored such arithmological treatises as *De principiis numerorum* and *Hebdomades*. His other works are replete with arithmological references,⁴⁵ at least some of which he must have taken from *Anonymus Arithmologicus* – a pseudo-Pythagorean treatise, which Frank Robbins dated to the late second or early first century BCE.⁴⁶ It is difficult – if at all possible – to trace Varro's source for the definition in question, but it certainly was influenced by the Neopythagorean movement and contained a lot of arithmological material.

⁴³ Cato, *Fest.* 258. 20; id. *RR* 57. 2; Plaut. *Curc.* 1. 2. 15; Plin. *NH* 14. 14. 16.

⁴⁴ Cass. *In Psalm.* 8. 275; Fav. *Eul.* 15. 2, 6; *TLL* s.v. *cubus*.

⁴⁵ See Palmer 1970, 5–34.

⁴⁶ Robbins 1920, 309–322.

c) Line

‘Linea’ autem a nostris dicitur, quam γραμμὴν Graeci nominant. Eam M. Varro ita definit: “Linea est”, inquit, “longitudo quaedam sine latitudine et altitudine”. Εὐκλείδης autem brevius praetermissa altitudine: “γραμμή”, inquit, “est μήκος ἀπλατέζ”, quod exprimere uno Latine verbo non queas, nisi audeas dicere ‘inlatabile’.

Varro defines line as “some length without breadth or height”. The comparison to Euclid’s definition follows, with Varro noting that Euclid omits “height”, saying “γραμμὴ μήκος ἀπλατέζ” (= *Euc. Def.* 1. 2). Was it Varro (or Varro’s source), or was it Gellius himself to draw this comparison? Based on the nature of Varro’s suggested source (or sources), it most likely contained the reference to Euclid in the first place.

It is quite obvious that Varro’s definition builds upon that of Euclid, expanding upon the latter by adding height. Very few line definitions take height into account, with only Hero (*Def.* 2. 1), Proclus (*In Eucl.* 97. 2–3),⁴⁷ and Macrobius (*Somn.* 2. 2. 5) doing so in addition to Varro.

Hero’s *Definitiones*⁴⁸ belong to a vast body of works that can be described as an “Introduction to the Elements”. The need for this type of treatise arises when knowledge of the basics of arithmetic and geometry becomes a *sine qua non* not only for people in the fields directly connected to the exact sciences (i. e. architecture and land surveying), but also for well-educated people in general. Consequently, such introductions were needed and used in education (e. g. in books by Pappus and Proclus).

In the *Prooemium*, Hero promises to his addressee Dionysius “a sketch of the technical terms premised in the elements of geometry”.⁴⁹ His whole arrangement is based “upon the teaching of Euclid, the writer of the elements of theoretical geometry”. Hero aims to give his reader “a good general understanding not only of Euclid’s works, but of many others pertaining to geometry”. To these ends, Hero provides each term with a number of definitions to accommodate for a deeper understanding of the subject matter. Hero’s treatment of the term ‘line’ is a good example of this approach (*Def.* 2):

⁴⁷ This definition might go down to Geminus (see Tittel 1895, 20).

⁴⁸ Hero’s authorship of *Definitiones* was put into question by Knorr, who argues that this work is due to Diophantus (Knorr 1993, 180–192). V. contra: Giardina 2003, 83–85. Cuomo attributes *Definitiones* to Hero, at the same time acknowledging that the treatise might be due to Diophantus and that the question of authorship may ultimately remain unsolved (Cuomo 2001, 161, 163).

⁴⁹ Tr. Thomas 1957, 467.

Line is length without breadth and without depth or what first takes existence in magnitude or what has one dimension and is divisible as well; it originates when a point flows from up downwards according to the notion of continuum, and is surrounded and limited by points, itself being the limit of a surface. One can say that a line is what divides the sunlight from the shadow or the shadow from the lighted part and in a toga imagined as a continuum <it divides> the purple line from the wool or the wool from the purple. Already in customary language we have an idea of the line as having only length, but neither breadth nor depth. We say then: a wall is according to hypothesis 100 cubits, without considering the breadth or the thickness, or a road is 50 stades, only the length, without also concerning ourselves with its breadth, so that the calculation of that as well is for us linear; it is in fact also called linear measurement.⁵⁰

Hero lived in the first century CE, and he most certainly could not be Varro's source; still, his *Definitiones* are a representative example of a tradition far preceding both his own time and that of Varro. Thus, using his *Definitiones* as an example, we now have a clearer picture of what sources Varro might have used in putting together the books on the *quadrivium*: among his sources, there definitely were some Hellenistic introductions to μαθήματα. These introductions were meant to elucidate the concepts succinctly defined in Euclid, but some of them were likely not limited to Euclid: traces of the post-Euclidean tradition are found in Hero and Geminus,⁵¹ while some mathematical extracts from the commentary to *Theaetetus* do not follow Euclid.⁵²

Parts of geometry

Book sixteen of *Noctes Atticae* contains yet another fragment on geometry, this time with Varro's notes on the state of Roman education in exact sciences.

a) Optics

Pars quaedam geometriae ὀπτική appellatur, quae ad oculos pertinet, pars altera, quae ad auris, κανονική vocatur, qua musici ut fundamento artis suae utuntur. Utraque harum spatiis et intervallis linearum et ratione numerorum constat.

⁵⁰ For commentary, see Giardina 2003, 265–270.

⁵¹ Tittel 1912, 1049–1050.

⁵² Cuomo 2001, 143–145.

Ὀπτική facit multa demiranda id genus, ut in speculo uno imagines unius rei plures appareant; item ut speculum in loco certo positum nihil imaginet, aliorum translatum faciat imagines; item si rectus speculum spectes, imago fiat tua eiusmodi, ut caput deorsum videatur, pedes sursum. Reddit etiam causas ea disciplina, cur istae quoque visiones fallant, ut quae in aqua conspiciuntur, maiora ad oculos fiant, quae procul ab oculis sunt, minora (Gell. *NA* 16. 18).

A part of geometry which relates to the sight is called optics, another part, relating to the ears, is known as canonic, which musicians make use of as the foundation of their art. Both of these rely on the spaces and the intervals between lines and on number ratios.

Optics affect many surprising things, such as the appearance in one mirror of several images of the same thing; also that a mirror placed in a certain position shows no image, but when moved to another spot gives reflections; also that if you look straight into a mirror, your reflection is such that your head appears below and your feet uppermost. This science also gives the reasons for optical illusions, such as the magnifying of objects seen in the water, and the small size of those that are remote from the eye.⁵³

Ancient optics is a mathematical visual ray theory. Aristotle includes optics among the “more physical” of the mathematical sciences (*Ph.* 2. 2. 194 a 7–12), along with harmonics and astronomy. Since Aristotle, optics had been considered a science subordinate to geometry. It aimed to explain visual perceptions of space, perspective, and visual illusions. Catoptrics, i. e. the study of reflection and refraction, was built upon the same geometrical laws as optics, but it quite early became an independent discipline (probably with Euclid).⁵⁴

In our text, optics is defined as a part of geometry relating to the eyes which builds upon spaces and intervals between lines and number ratios. Note that in our text, optics is a part of geometry, not a subordinate science. The next paragraph exemplifies how optics is used. The majority of examples are not optics proper, but rather catoptrics. Mirror images are followed by an example of refraction (the magnification of objects seen in water) and one optical example of perspective that is very basic in nature: remote objects appear small. Thus, the source must have covered all varieties of ancient optics and catoptrics (mirror images and illusions, refraction, perspective), but the nature of the original treatise is hard to

⁵³ Tr. Rolfe 1927, 187, 189, with modifications.

⁵⁴ Lejeune 1957, 180.

identify: was it a scientific treatise or an educational one? One thing has to be noted though: in the text, optics is based solely on mathematics, and physical (i. e. pertaining to natural philosophy) ἀρχαί are not mentioned – meaning that optics, in addition to geometry, is independent from natural philosophy.⁵⁵

The outline of optics in our text is pretty simplistic. It does not cover even the basics of the science: spaces and intervals between the lines are referred to, while the visual cone itself, on which the very principles of optics are based, is left out. It is unclear which one of the three ancient physical vision theories was adopted here.⁵⁶ Instead, the text concentrates on curious cases – *multa demiranda* – that are probably meant to provoke the reader's interest and entice them to learn more about optics. Such a disproportional amount of 'edutaining' content might well be due to Gellius's editorial choices: Varro could have covered the basics of all optical disciplines in equal measure.

Identifying Varro's sources for optics seems a futile endeavor: the text provides us with the most basic and unoriginal information present in any optical treatise (i. e. the small size of objects remote from the eye). Catoptrics, on the other hand, is described in greater detail. Unfortunately, our sources on catoptrics are in a lamentable state. Pseudo-Euclid's *Catoptrics* is a late compilation authored most probably by Theon of Alexandria⁵⁷ (although it is possible that it contains some traces of original Euclid's *Catoptrics*).⁵⁸ Meanwhile, Hero of Alexandria's *Catoptrics* survives in a Medieval Latin translation of a poorly-preserved Greek text.⁵⁹ Books three, four, and five of Ptolemy's *Optics* are our third and last source on catoptrics, and are also a Medieval Latin translation – this time from Arabic – with the fifth book incomplete. Our text, on the other hand, does not provide us with enough scientific data that would allow us to determine its place in the development of catoptrics. The magnification of objects seen in water had been known since at least Archimedes,⁶⁰ and the mirror reflections described by Gellius are also not unique to any other catoptrical treatise.

⁵⁵ Cf. Posidonius: *quae causa in speculo imagines exprimat sciet sapiens: illud tibi geometres potest dicere, quantum abesse debeat corpus ab imagine et qualis forma speculi quales imagines reddat* (Sen. *Epist.* 88. 27 = F 90 E–K).

⁵⁶ On physical theories of vision see Thibodeau 2016, 130–144.

⁵⁷ Heiberg 1882, 148 ff.

⁵⁸ See Lejeune 1957.

⁵⁹ For references, see Lejeune 1957, 5.

⁶⁰ Lejeune 1957, 176–179.

b) Harmonics and mathematical education

(*Text continues*) Κανονική autem longitudes et altitudines vocis emetitur. Longior mensura vocis ῥυθμός dicitur, altior μέλος. Est et alia species, quae appellatur μετρική, per quam syllabarum longarum et brevium et mediocrium iunctura et modus congruens cum principiis geometriae aurium mensura examinatur. “Sed haec”, inquit M. Varro, “aut omnino non discimus aut prius desistimus, quam intellegamus, cur discenda. Voluptas autem”, inquit, “vel utilitas talium disciplinarum in postprincipiis existit, cum perfectae absolutaeque sunt; in principiis vero ipsis ineptae et insuaves videntur” (Gell. *NA* 16. 18).

Canonics, on the other hand, measures the durations and pitches of sounds. The measure of the duration of sounds is called ῥυθμός, and the measure of their pitch is called μέλος. There is also another variety of canonics which is called metric, by which the combination of long and short syllables, and those which are neither long nor short, and the verse measure according to the principles of geometry are examined with the aid of the ears. “But these things”, says Marcus Varro, “we either do not learn at all, or we leave off before we know why they ought to be learned. But the pleasure”, he says, “and the advantage of such sciences appear in their later study, when they have been completely mastered; but in their mere elements they seem foolish and unattractive”.⁶¹

Ancient harmonics existed in two versions: mathematical and empirical. The first stems from Pythagoreans, who discovered certain mathematical ratios of concordant intervals. Metaphysically, these relations provided a model for a “harmonious” universe. The empirical branch of harmonics meanwhile first appeared among practical musicians and was further developed by Aristoxenus of Tarentum in the late fourth century BCE.⁶² Aristoxenus dismissed the mathematical approach and insisted that music exists only in the domain accessible to hearing. Harmonics is commonly viewed as a science subordinate to arithmetic, not as a part of geometry.

According to Gellius, harmonics is a part of geometry related to the ears, which, in addition to optics, is based upon the spaces and intervals between lines and on number ratios.⁶³ Another musical discipline is metrics, “by which the combination of long and short syllables, and those which are neither long nor short, and the verse measure according

⁶¹ Tr. Rolfe 1927, 189, with modifications.

⁶² Barker 2018, 428–448.

⁶³ He uses Hellenistic term ‘canonics’, which suggests a Hellenistic source.

to the principles of geometry are examined with the aid of the ears". Fortunately, elsewhere in Gellius, we have an example of what is meant by geometrical principles applied to the verse meters (Gell. *NA*. 18. 15. 2):

M. etiam Varro in libris disciplinarum scripsit observasse sese in versu hexametro, quod omnimodo quintus semipes verbum finiret et quod priores quinque semipedes aequae magnam vim haberent in efficiendo versu atque alii posteriores septem, idque ipsum ratione quadam geometrica fieri disserit.⁶⁴

Varro might be unique in presenting harmonics as a part of geometry,⁶⁵ but we have to be very careful in drawing far-reaching conclusions from this fact. We have already seen that Varro uses the term 'geometry' extremely generously, as an umbrella term for μαθήματα.

Let us look whether it is possible to place our fragment inside different harmonical approaches. On the one hand, it certainly does have some features of Pythagorean harmonics, as the numerical ratios are used to denote intervals between notes. More surprising features include *ratio geometrica* in verses, contrasted with a feature of an empirical approach, i. e. *aurium mensura*. A more elaborate classification of different harmonic approaches exists: Ptolemais of Cyrene – placed by Andrew Barker in the first century BCE – identifies five groups of harmonic theorists based on the roles assigned to reason and perception (Porph. *Harm.* 25. 3 – 26. 5).⁶⁶ One of the five groups consists of those who favor reason, but allow perception an auxiliary role. Ptolemais ascribes this approach to some of the Pythagoreans. Didymus characterizes the same group as follows (Porph. *Harm.* 26. 18–24):

They adopt [from perception] certain kindling sparks ... and construct the theorems that are put together out of them through reason on its own, taking no further notice of perception. Hence on occasions when only what follows rationally is carefully preserved, and perception bears witness against it, it is possible for them to be not in the least disturbed by this sort of discord, but to pin their faith upon reason and dismiss perception as going astray.⁶⁷

⁶⁴ This fragment is further elucidated by Augustine (*De musica* 5. 12. 26), see Holford-Strevens 1994, 483–486.

⁶⁵ Creese 2010, 225.

⁶⁶ Barker 2018, 441.

⁶⁷ Tr. Barker 2018, 242–243.

This approach seems to be a good match for the one adopted by Varro.

Varro's educational comment on the lamentable state of mathematical education is congruent with everything we have seen so far: during his time, Roman mathematical education was pretty basic. It stopped well before students were able to enjoy the fruits of such an education. Varro's book was directed at those who had already completed the standard school curriculum and wanted to study rhetoric or philosophy. *Disciplinae* provided them with the very basics of each subject and introduced them to some of the most famous scientific discoveries. The practical character of the book was ensured by the sources that Varro used, i.e. various introductions to μαθήματα. However, the question of whether the book was used for self-education or in class depends not solely on the book itself, but also on the state of post-school education in Varro's time. As seen in the example of Diodotus in the introduction to this article, mathematical education was private, which means that each tutor would choose study materials according to the students' needs and abilities. Varro's book, though very basic in nature, must have been quite popular with beginners, as it probably was at the time the only book in Latin to cover the fundamentals of all disciplines, making it more accessible to readers.

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This article brings together the evidence concerning the subjects of the quadrivium in Varro’s *Disciplines* and provides a description of the book’s content, composition, and sources, while at the same time discussing the level of post-school mathematical education in Varro’s time. Polarizing views exist on the matter of post-school mathematical education, with some scholars placing it as early as the beginning of the Hellenistic period, and others linking its emergence to Neoplatonic circles in the fourth century CE. I argue that it is possible to attest to the existence of post-school mathematical education in the first century BCE, even though it was pretty basic in nature and did not go beyond the fundamentals of the subjects of the quadrivium, as the contents of Varro’s book suggest. The first section of the article covers Varro’s unconventional views on the origin of geometry: Varro rejects the Egyptian origin of geometry and traces its invention

back to the dawn of human civilization. The second section deals with Varro's geometrical definitions and their relation to the Euclidean tradition, showing that among his sources, there definitely were some Hellenistic introductions to μαθήματα. The final section focuses on Varro's conception of optics and canonics; here, his approach to canonics is identified as mostly mathematical with some empirical features.

В статье собраны свидетельства о предметах квадривиума в сочинении Варрона *Disciplinae*, дается описание его содержания, композиции и источников, а также обсуждается уровень послешкольного математического образования во времена Варрона. В научной среде существуют противоположные взгляды на проблему послешкольного математического образования: некоторые ученые относят его возникновение к началу эллинистического периода, а другие связывают его появление с неоплатоническими кругами в четвертом веке нашей эры. На деле существование послешкольного математического образования можно засвидетельствовать в первом веке до нашей эры, хотя оно было, в сущности, базовым и не выходило за рамки основ предметов квадривиума, как предполагает содержание книги Варрона. Первая часть статьи посвящена нетрадиционным взглядам Варрона на происхождение геометрии: Варрон отвергает традицию о египетском происхождении геометрии и относит ее изобретение ко времени возникновения человеческой цивилизации. Второй раздел посвящен геометрическим определениям Варрона и их связи с евклидовой традицией. В нем показано, что среди его источников определенно были некоторые эллинистические введения в μαθήματα. Заключительный раздел посвящен концепции оптики и каноники у Варрона: здесь его подход к канонике определяется как в основном математический с некоторыми эмпирическими особенностями.

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